

Student Name \_\_\_\_\_

Teacher Name \_\_\_\_\_

School \_\_\_\_\_

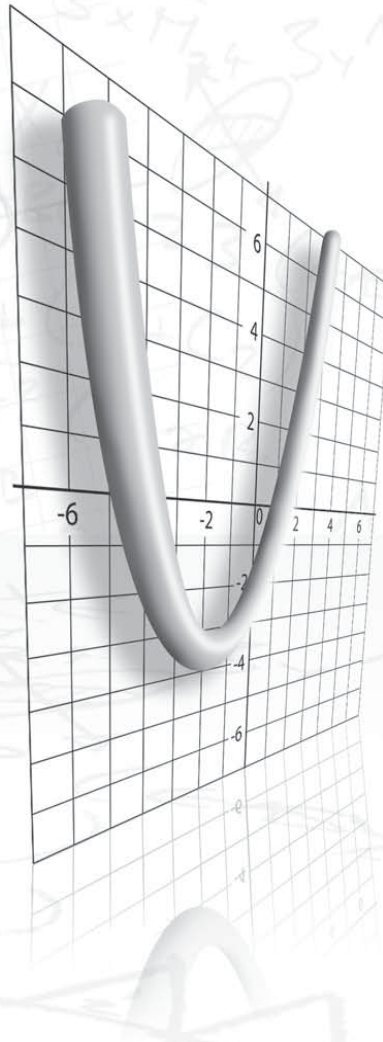
System \_\_\_\_\_

# ALGEBRA II

## Item Sampler

**Tennessee End of Course Assessment  
Algebra II Form 1**

**Reporting Category 1: Mathematical Processes**

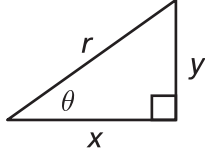


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## Algebra II Reference Page

Trigonometric Functions	
$\sin \theta = \frac{y}{r}, \quad \csc \theta = \frac{r}{y}$ $\cos \theta = \frac{x}{r}, \quad \sec \theta = \frac{r}{x}$ $\tan \theta = \frac{y}{x}, \quad \cot \theta = \frac{x}{y}$	 $r = \sqrt{x^2 + y^2}$

Logarithm Properties
$\log_b MN = \log_b M + \log_b N$ $\log_b \left( \frac{M}{N} \right) = \log_b M - \log_b N$ $\log_b M^p = p \log_b M$ $\log_b x = y \Leftrightarrow x = b^y$

Arithmetic and Geometric Sequences and Series	
$a_1 = 1^{\text{st}} \text{ term}$ $r = \text{common ratio}$ $d = \text{common difference}$ $a_n = n^{\text{th}} \text{ term}$ $n = \text{number of terms in series}$	
Arithmetic Sequence: $a_n = a_1 + (n-1)d$	Geometric Sequence: $a_n = a_1 r^{n-1}$
Sum of a Finite Arithmetic Series:	$S_n = \frac{n(a_1 + a_n)}{2}$ or $S_n = \frac{1}{2}n[2a_1 + (n-1)d]$
Sum of a Finite Geometric Series:	$S_n = \frac{a_1(1-r^n)}{1-r}, \quad r \neq 1$
Sum of an Infinite Geometric Series:	$S = \frac{a_1}{1-r}$ where $ r  < 1$

Combinations
${}_nC_r = \frac{n!}{r!(n-r)!}$

Permutations
${}_nP_r = \frac{n!}{(n-r)!}$

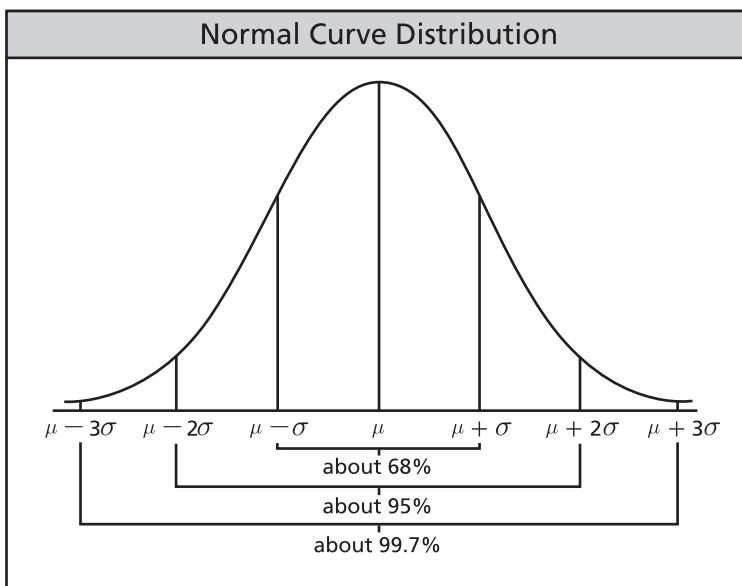
Binomial Theorem
$(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$

Quadratic Formula
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $y = ax^2 + bx + c$

Interest Formulas	
Compound interest: $A = P \left( 1 + \frac{r}{n} \right)^{nt}$	$P = \text{present value}$
Continuous compound interest: $A = Pe^{rt}$	$A = \text{future value}$
	$r = \text{annual interest rate}$
	$t = \text{time in years}$
	$n = \text{frequency of compounding per year}$

## Algebra II Reference Page

Conic Sections – Standard Equations			
Parabola	$y = a(x - h)^2 + k$	or	$x = a(y - k)^2 + h$
	$(y - k)^2 = 4p(x - h)$	or	$(x - h)^2 = 4p(y - k)$
Circle	$(x - h)^2 + (y - k)^2 = r^2$		
Ellipse	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$	or	$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$
Hyperbola	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$	or	$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$



**Standard Deviation**

The standard deviation,  $\sigma$ , for values  $x_1, x_2, x_3, \dots, x_n$  with mean  $\mu$  is determined by the following:

$$\sigma = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \dots + (x_n - \mu)^2}{n}}$$

**Probability Formulas**

Exclusive  
 $P(A \text{ or } B) = P(A) + P(B)$

Inclusive  
 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Independent  
 $P(A \text{ and } B) = P(A) \cdot P(B)$

Dependent  
 $P(A \text{ and } B) = P(A) \cdot P(B|A)$

Conditional  
 $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$

## Algebra II Reference Page

### Cramer's Rule for Solving a System of Linear Equations

For a  $2 \times 2$  System:

$$\begin{array}{l} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{array} \quad x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

For a  $3 \times 3$  System:

$$\begin{array}{l} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{array} \quad x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \quad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

### Converting Degrees to Radians

Multiply degree measure  
by  $\frac{\pi}{180^\circ}$

### Converting Radians to Degrees

Multiply radian measure  
by  $\frac{180^\circ}{\pi}$

### Definition of "i"

$$\begin{aligned} i^2 &= -1 \\ i &= \sqrt{-1} \end{aligned}$$

### Absolute Value of a Complex Number

$$|a + bi| = \sqrt{a^2 + b^2}$$

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## **Introduction to Algebra II**

### **Content of tests**

The testing program titled the *Tennessee End of Course Assessment* was established to meet the Tennessee mandate for end of course assessments in Tennessee secondary schools. These tests measure the Tennessee State Performance Indicators. Subject areas covered by the end of course assessments include Mathematics, Language Arts, History, and Science.

### **Test development**

For the *Tennessee End of Course Assessment*, a staff of writers – composed of both teachers and professional test developers experienced in each of the content areas – researched and wrote the items. Professional editors and content specialists carefully reviewed all items and test directions for content and accuracy. To provide a large pool of items for final test selection, the test developers created approximately twice as many items as were needed in the final editions of the tests.

After tryout tests were administered, student responses were analyzed. Professional content editors and researchers carefully reviewed items, their data, and test directions for content, suitability, and accuracy before including particular items and test directions in operational tests.

## Test administration

*Tennessee End of Course Assessments* are given to students as they near the end of courses that are included in the program. Tests may be given midyear for block schedules or at the end of the school year.

You will have ample time to read and answer each of the questions. The Algebra II test has been designed to be administered in one session and is not timed. The first 15 minutes are set aside to complete identifying data on the answer sheet.

Calculator use is optional. Sharing calculators during testing is not permitted.

The following types of calculators/devices may **NOT** be used during the test:

- pocket organizers
- electronic writing pads or input devices
- Some examples of prohibited calculators are:
  - Casio models: CFX-9970G, Algebra FX 2.0
  - Hewlett-Packard models: HP-40G, HP-49G
  - Texas Instruments models: TI-89, TI-92, Voyage 200, TI-NSPIRE – the CAS version (The non-CAS version of TI-NSPIRE is allowable.)
- calculators that can communicate (transfer data or information) wirelessly with other student calculators/devices
- cell phones, PSPs, and/or iPods
- Students may use any four-function, scientific, or graphing calculator does not have any of the above features. The use of units that have a Computer Algebra System (CAS) is NOT allowed.



## **Tips for Taking the Test**

### **Preparing for the test**

- Review this Tennessee End of Course Item Sampler for Algebra II carefully and thoroughly.
- Acquire the Tennessee End of Course Practice Test for Algebra II, and take the test several times.
- Become familiar with the correct way to mark answers on the answer sheet.

### **Before the test**

- Get a good night's sleep. To do your best, you need to be rested.

### **During the test**

- Relax. It is normal to be somewhat nervous before the test. Try to relax and not worry.
- Listen. Listen to and read the test directions carefully. Ask for an explanation of the directions if you do not understand them.
- Plan your time. Do not spend too much time on any one question. If a question seems to take too long, skip it and return to it later. First answer all questions that you are sure about.
- Think. If you are not sure how to answer a question, read it again and try your best to answer the question. Rule out answer choices that you know are incorrect and choose from those that remain.

## **Directions for Using the Item Sampler**

This Item Sampler for Algebra II provides specific information to students and teachers. It contains examples of different item types for each Performance Indicator that may be tested in any given end of course test administration. Performance Indicators have been grouped by Reporting Categories. These Reporting Categories will be used to report information regarding performance on the end of course test to students, teachers, schools, and systems.

The items in this Item Sampler will not be found in the end of course tests. The number of items in this Item Sampler does not reflect the emphasis of content on the test. In order to identify the emphasis of content, the End of Course Assessment Practice Test for Algebra I should be used. The Practice Test gives a better representation of content emphasis across Reporting Categories and Performance Indicators.

An Answer Key is located in Page 23. Use it to check your answers. Review items that you get wrong.

## Reporting Category: Mathematical Processes

Numbers 1 through 23

**Performance Indicator:** 3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.

1.

Which table of values **best** represents the equation below?

$$y = 4(0.6)^x$$

☐ A

x	y
0	0
0.5	1.2
1	2.4
1.5	3.6
2	4.8

☐ C

x	y
0	1
0.5	0.77
1	0.6
1.5	0.46
2	0.36

☐ B

x	y
0	1
0.5	1.55
1	2.4
1.5	3.72
2	5.76

☐ D

x	y
0	4
0.5	3.1
1	2.4
1.5	1.86
2	1.44

**Performance Indicator:** 3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.

2.

Which table of values **best** represents the function below?

$$f(x) = \log(x - 1)^2$$

☐ A

x	f(x)
2	0
3	0.09
4	0.23
5	0.36
6	0.49

☐ C

x	f(x)
2	0
3	0.60
4	0.95
5	1.20
6	1.40

☐ B

x	f(x)
2	0
3	0.15
4	0.24
5	0.30
6	0.35

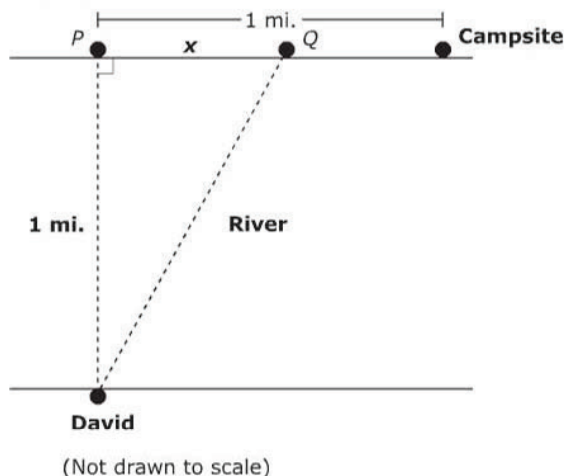
☐ D

x	f(x)
2	0
3	0.30
4	0.48
5	0.60
6	0.70

**Performance Indicator:** 3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.

3.

David is standing on the bank of a one-mile-wide river. He has to return to his campsite on the opposite side of the river. He swims across the river to Point Q at 2 miles per hour and walks the remaining distance to the campsite at 3 miles per hour. The campsite is one mile from Point P.



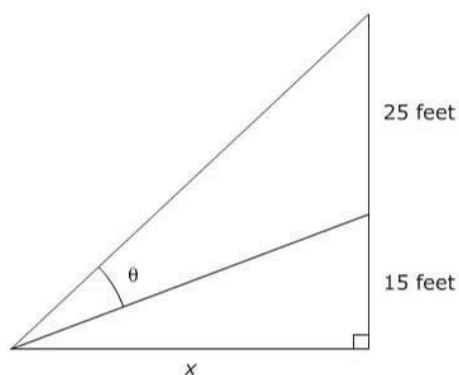
If  $x$  is the distance between Point P and Point Q, which equation represents David's total time of travel,  $T$ ? ( $t = \frac{d}{r}$ )

- ☐ A  $T = \frac{\sqrt{1-x^2}}{2} + \frac{1-x}{3}$
- ☐ B  $T = \frac{\sqrt{1+x^2}}{3} + \frac{1-x}{2}$
- ☐ C  $T = \frac{\sqrt{1+x^2}}{2} + \frac{1-x}{3}$
- ☐ D  $T = \frac{\sqrt{1-x^2}}{3} + \frac{1-x}{2}$

**Performance Indicator:** 3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.

4.

The picture below shows a 25-foot movie screen located 15 feet above the floor.



If Alex is standing at a distance of  $x$  feet from the screen, which expression represents the viewing angle  $\theta$ ?

- ☐ A  $\theta = \tan^{-1}\left(\frac{25}{x}\right) + \tan^{-1}\left(\frac{15}{x}\right)$
- ☐ B  $\theta = \tan^{-1}\left(\frac{25}{x}\right) - \tan^{-1}\left(\frac{15}{x}\right)$
- ☐ C  $\theta = \tan^{-1}\left(\frac{40}{x}\right) + \tan^{-1}\left(\frac{15}{x}\right)$
- ☐ D  $\theta = \tan^{-1}\left(\frac{40}{x}\right) - \tan^{-1}\left(\frac{15}{x}\right)$

**Performance Indicator:** 3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.

5.

Which equation **best** represents the data given in the table below?

$x$	$y$
1	0
2.5	4.6
4	6.9
5.5	8.5

- ☐ A  $y = 5 \ln x$
- ☐ B  $y = -0.5x^2 + 4.8x - 4.3$
- ☐ C  $y = 3.07x - 3.07$
- ☐ D  $y = 1.26(2^x) - 2.52$

**Performance Indicator:** 3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.

6.

Which equation **best** represents the relationship between the  $x$  and  $y$  values in the table below?

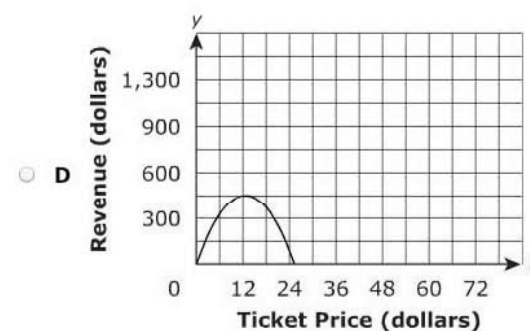
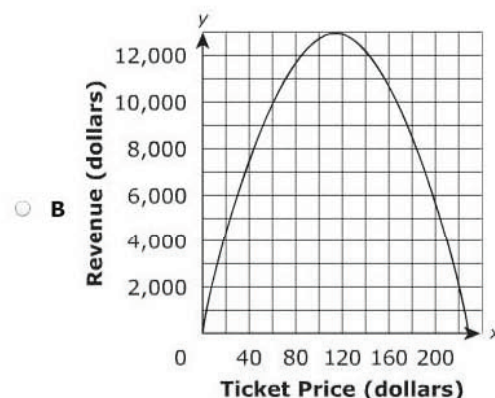
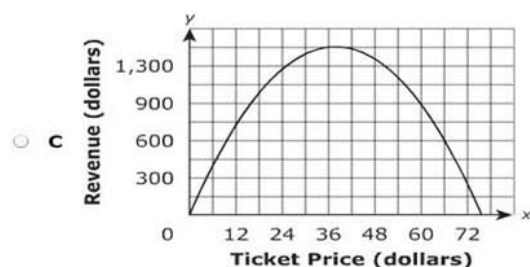
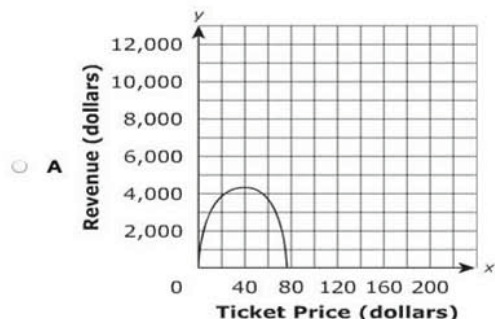
$x$	$y$
0	0.4
0.2	0.5
0.4	0.6
0.6	0.8
0.8	1.0
1	1.2

- ☐ A  $y = 0.3(x^4)$
- ☐ B  $y = 0.4(x^3)$
- ☐ C  $y = 0.4(3^x)$
- ☐ D  $y = 0.3(4^x)$

**Performance Indicator:** 3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.

7.

The function  $R(x) = 225x - 3x^2$  models the revenue generated from a concert, where  $x$  is the price of a ticket. Which graph **best** represents this function?

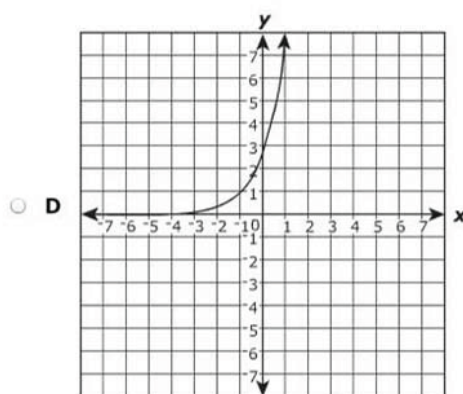
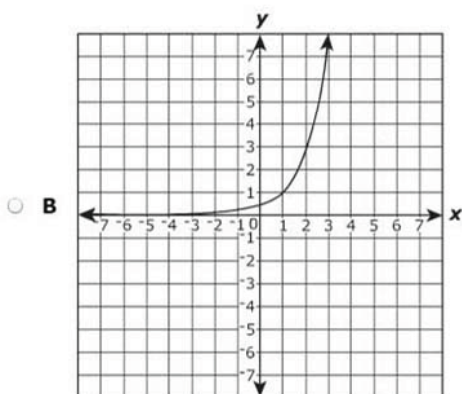
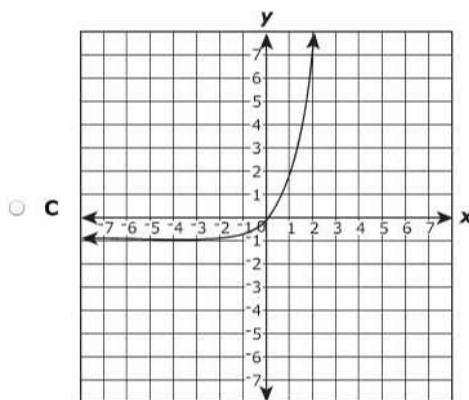
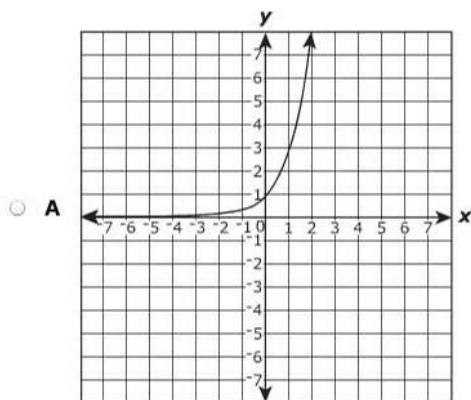


**Performance Indicator:** 3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.

8.

Which graph models the equation given below?

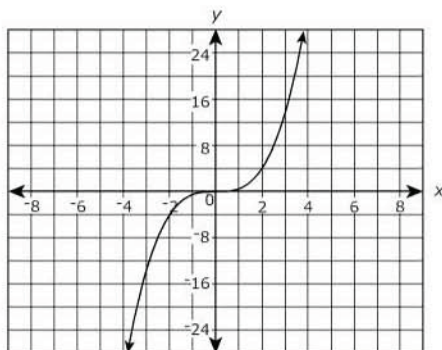
$$y = 3^{x-1}$$



**Performance Indicator:** 3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.

9.

Which table of values best represents the graph below?



☐ A

$x$	$f(x)$
-4	-32
-2	-4
0	0
2	4

☐ C

$x$	$f(x)$
-3	-12
-1	0
1	0
3	12

☐ B

$x$	$f(x)$
-32	-4
-4	-2
0	0
4	2

☐ D

$x$	$f(x)$
-12	-3
0	-1
0	1
12	3



**Performance Indicator:** 3103.1.2 Recognize and describe errors in data collection and analysis as well as identifying representations of data as being accurate or misleading.

10.

A dairy farm wants to conduct a statewide survey on the popularity of a new dairy product 3 months after its launch. Which of these samples would be the least biased?

- ☐ A every employee of the dairy farm in the state
- ☐ B every resident of a particular town in the state
- ☐ C every distributor for the dairy farm in the state
- ☐ D every grocery store owner in a particular town in the state

**Performance Indicator:** 3103.1.2 Recognize and describe errors in data collection and analysis as well as identifying representations of data as being accurate or misleading.

11.

The frequency table below shows how the participants from last year's race improved their times in this year's race.

**Time Improvements for Race**

Improvements (seconds)	Frequency
0–4	12
5–9	22
10–14	20
15–19	18
21–29	36
31–39	28
40–44	8

Which statement about the frequency table is true?

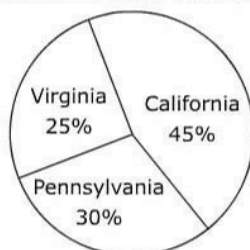
- ☐ A The table is accurate because the times are expressed in seconds.
- ☐ B The table is accurate because a large number of participants are included.
- ☐ C The table is misleading because the intervals in the left column are not uniform.
- ☐ D The table is misleading because the improvements should be ordered from greatest to least.

**Performance Indicator:** 3103.1.2 Recognize and describe errors in data collection and analysis as well as identifying representations of data as being accurate or misleading.

12.

Jamie manages a factory that uses 45 tons of raw materials each month. She calculates that 18 tons are shipped from California, 12 tons are shipped from Pennsylvania, 10 tons are shipped from Virginia, and the remaining raw materials are imported from another country. Jamie makes the circle graph below to represent the sources of raw materials.

**Sources of Raw Materials**



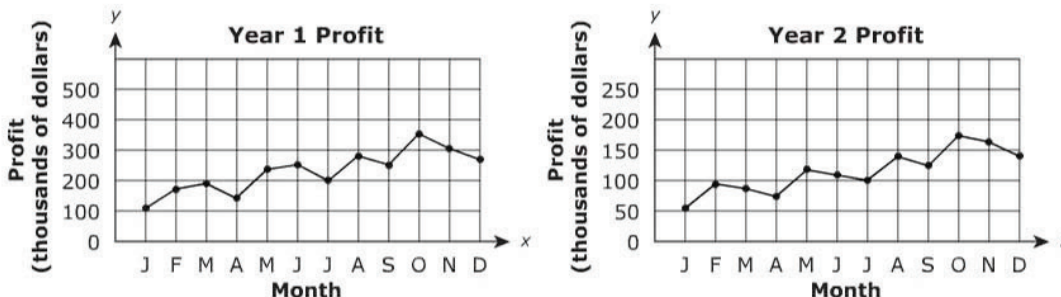
Which statement regarding the graph is true?

- ☐ A The graph is accurate because the weights represent the correct percents of the total weight of the raw materials.
- ☐ B The graph is misleading because the data represented in the circle graph does not represent the whole.
- ☐ C The graph is misleading because the percents shown in the graph do not add up to 100%.
- ☐ D The graph is accurate because a circle graph best represents the data.

**Performance Indicator:** 3103.1.2 Recognize and describe errors in data collection and analysis as well as identifying representations of data as being accurate or misleading.

13.

A company publishes line graphs showing the profit for two different years. The company claims that the profit for Year 2 is about the same as the profit for Year 1.



Which is the best explanation of why the company's statement is misleading?

- ☐ A The data are connected by line segments.
- ☐ B The vertical scales on the graphs are different.
- ☐ C The horizontal scales are in months instead of weeks.
- ☐ D The vertical scales are in thousands of dollars instead of hundreds of dollars.

**Performance Indicator:** 3103.1.3 Use technology tools to identify and describe patterns in data using nonlinear and transcendental functions that approximate data as well as using those functions to solve contextual problems.

14.

The table below shows the power used by the regulator of a fan.

Voltage (volts)	Power (watts)
5	0.31
6	0.29
7	0.26
11	0.11
12	0.02

The data is modeled by a quadratic equation. What is the power used by the regulator, to the nearest hundredth watt, when the voltage is 9 volts?

- ☐ A 0.04 watt
- ☐ B 0.19 watt
- ☐ C 0.20 watt
- ☐ D 0.24 watt

**Performance Indicator:** 3103.1.3 Use technology tools to identify and describe patterns in data using nonlinear and transcendental functions that approximate data as well as using those functions to solve contextual problems.

15.

A certain type of bacteria grows exponentially. The number of bacteria at the end of every 5 hours is shown in the table.

Number of Bacteria

Time (hours)	Number of Bacteria
0	100
5	300
10	900
15	2,700
20	8,096

Based on the data in the table, what would be the approximate number of bacteria at the end of 12 hours?

- ☐ A 580
- ☐ B 1,140
- ☐ C 1,396
- ☐ D 1,620

**Performance Indicator:** 3103.1.3 Use technology tools to identify and describe patterns in data using nonlinear and transcendental functions that approximate data as well as using those functions to solve contextual problems.

16.

The table below gives the atmospheric pressure, in kilograms per square meter ( $\text{kg}/\text{m}^2$ ), at certain altitudes, in kilometers.

**Atmospheric Pressure**

Altitude (kilometers)	Atmospheric Pressure (kilograms per square meter)
0	10,958
1	9,432
2	8,118
3	6,987
4	6,014

If atmospheric pressure decreases exponentially as a function of altitude, which is closest to the atmospheric pressure at an altitude of 2.5 kilometers?

- ☐ A 7,553  $\text{kg}/\text{m}^2$
- ☐ B 7,531  $\text{kg}/\text{m}^2$
- ☐ C 7,461  $\text{kg}/\text{m}^2$
- ☐ D 7,143  $\text{kg}/\text{m}^2$

**Performance Indicator:** 3103.1.3 Use technology tools to identify and describe patterns in data using nonlinear and transcendental functions that approximate data as well as using those functions to solve contextual problems.

17.

The table below gives the length of time,  $t$ , for a certain variety of insects to grow to a population,  $p$ .

**Population of Insects**

Time (days), $t$	Number of Insects, $p$
0	12
7	21
20	59
23	76
26	96

The data are modeled by an exponential equation. Based on these results, which is closest to the expected value of  $p$  when  $t = 10$ ?

- ☐ A 24
- ☐ B 27
- ☐ C 30
- ☐ D 37

**Performance Indicator:** 3103.1.4 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely to effectively communicate reasoning in the process of solving problems via mathematical modeling with both linear and nonlinear functions.

18.

Mark solved the problem as shown below. In which step did Mark's first mistake appear?

$$\sqrt{x} - 2 = 15$$

Step 1:  $\sqrt{x} = 15 + 2$

Step 2:  $(\sqrt{x})^2 = (\sqrt{17})^2$

Step 3:  $x^2 = 17$

Step 4:  $x = \sqrt{17}$

- ☐ A Step 1
- ☐ B Step 2
- ☐ C Step 3
- ☐ D Step 4

**Performance Indicator:** 3103.1.4 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely to effectively communicate reasoning in the process of solving problems via mathematical modeling with both linear and nonlinear functions.

19.

Jeremy makes the following statement.

"The domain of  $y = \sqrt{x - 4}$  is the set of all real nonnegative numbers and the range is the set of all real nonnegative numbers."

What produces a counterexample to this statement?

- ☐ A  $x = 0$
- ☐ B  $x = 4$
- ☐ C  $x = 4\pi$
- ☐ D  $x = \sqrt{20}$

**Performance Indicator:** 3103.1.4 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely to effectively communicate reasoning in the process of solving problems via mathematical modeling with both linear and nonlinear functions.

20.

Identify the step in which the error was made in solving for  $y$  below.

$$4y - 3 = -2[x - (-7)]$$

Step 1:  $4y - 3 = -2(x - 7)$

Step 2:  $4y - 3 = -2x + 14$

Step 3:  $4y = -2x + 17$

Step 4:  $y = \frac{1}{4}(-2x + 17)$

- ☐ A Step 1
- ☐ B Step 2
- ☐ C Step 3
- ☐ D Step 4

**Performance Indicator:** 3103.1.4 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely to effectively communicate reasoning in the process of solving problems via mathematical modeling with both linear and nonlinear functions.

21.

While analyzing the function  $f(x) = x^2$ , where  $x \geq 0$ , Nyako made the statement below.

"Every number in the range of the function is greater than or equal to the corresponding number in the domain."

Which value of  $x$  produces a counterexample to this statement?

- ☐ A  $x = 0$
- ☐ B  $x = \frac{1}{4}$
- ☐ C  $x = 1$
- ☐ D  $x = \sqrt{2}$



**Performance Indicator:** 3103.1.4 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely to effectively communicate reasoning in the process of solving problems via mathematical modeling with both linear and nonlinear functions.

22.

Albert performs the calculation below to determine the discriminant of the quadratic equation  $4x^2 - 3x + 1 = 0$  and to find the type of solutions.

Step 1:  $(-3)^2 - 4(4)(1)$

Step 2:  $-9 - 16$

Step 3:  $-25$

Step 4: Since the discriminant is less than 0, the equation has two imaginary roots.

Which is the first incorrect step?

- ☐ A Step 1
- ☐ B Step 2
- ☐ C Step 3
- ☐ D Step 4

**Performance Indicator:** 3103.1.4 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely to effectively communicate reasoning in the process of solving problems via mathematical modeling with both linear and nonlinear functions.

23.

Fabio used the procedure below to determine the domain of the function  $f(x) = \frac{x}{x^3 + x^2 - x + 15}$  for all real numbers  $x$ .

Step	Argument
$x^3 + x^2 - x + 15 \neq 0$	The function is not defined where the denominator is 0.
$(x + 3)(x^2 - 2x + 5) \neq 0$	Factorize
$x + 3 \neq 0$ and $x^2 - 2x + 5 \neq 0$	If $a \cdot b \neq 0$ , then $a \neq 0$ and $b \neq 0$ .
$x \neq -3$ and $x \neq \frac{2 \pm \sqrt{4 - 20}}{2}$	Using subtraction property of equality and the quadratic formula
Therefore, the domain of $f(x)$ does not include $x = -3$ .	?

Which argument did Fabio use to reach the conclusion?

- ☐ A  $\frac{2 \pm \sqrt{4 - 20}}{2}$  is a fraction, and therefore not real.
- ☐ B  $\frac{2 \pm \sqrt{4 - 20}}{2} = 1$  is positive, and therefore not real.
- ☐ C  $\frac{2 \pm \sqrt{4 - 20}}{2} = 1 \pm 2i$  is a complex number, and therefore not real.
- ☐ D  $\frac{2 \pm \sqrt{4 - 20}}{2} = 1 \pm 2i$  is positive, and therefore not real.

## Reporting Category 1: Mathematical Processes

Item Number	Correct Answer	Performance Indicator
1	D	3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.
2	C	3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.
3	C	3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.
4	D	3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.
5	A	3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.
6	C	3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.
7	A	3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.



8	B	3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.
9	A	3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of nonlinear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.
10	C	3103.1.2 Recognize and describe errors in data collection and analysis as well as identifying representations of data as being accurate or misleading.
11	C	3103.1.2 Recognize and describe errors in data collection and analysis as well as identifying representations of data as being accurate or misleading.
12	B	3103.1.2 Recognize and describe errors in data collection and analysis as well as identifying representations of data as being accurate or misleading.
13	B	3103.1.2 Recognize and describe errors in data collection and analysis as well as identifying representations of data as being accurate or misleading.
14	C	3103.1.3 Use technology tools to identify and describe patterns in data using nonlinear and transcendental functions that approximate data as well as using those functions to solve contextual problems.
15	C	3103.1.3 Use technology tools to identify and describe patterns in data using nonlinear and transcendental functions that approximate data as well as using those functions to solve contextual problems.
16	B	3103.1.3 Use technology tools to identify and describe patterns in data using nonlinear and transcendental functions that approximate data as well as using those functions to solve contextual problems.
17	B	3103.1.3 Use technology tools to identify and describe patterns in data using nonlinear and transcendental functions that approximate data as well as using those functions to solve contextual problems.

18	B	3103.1.4 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely to effectively communicate reasoning in the process of solving problems via mathematical modeling with both linear and nonlinear functions.
19	A	3103.1.4 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely to effectively communicate reasoning in the process of solving problems via mathematical modeling with both linear and nonlinear functions.
20	A	3103.1.4 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely to effectively communicate reasoning in the process of solving problems via mathematical modeling with both linear and nonlinear functions.
21	B	3103.1.4 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely to effectively communicate reasoning in the process of solving problems via mathematical modeling with both linear and nonlinear functions.
22	B	3103.1.4 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely to effectively communicate reasoning in the process of solving problems via mathematical modeling with both linear and nonlinear functions.
23	C	3103.1.4 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely to effectively communicate reasoning in the process of solving problems via mathematical modeling with both linear and nonlinear functions.

